

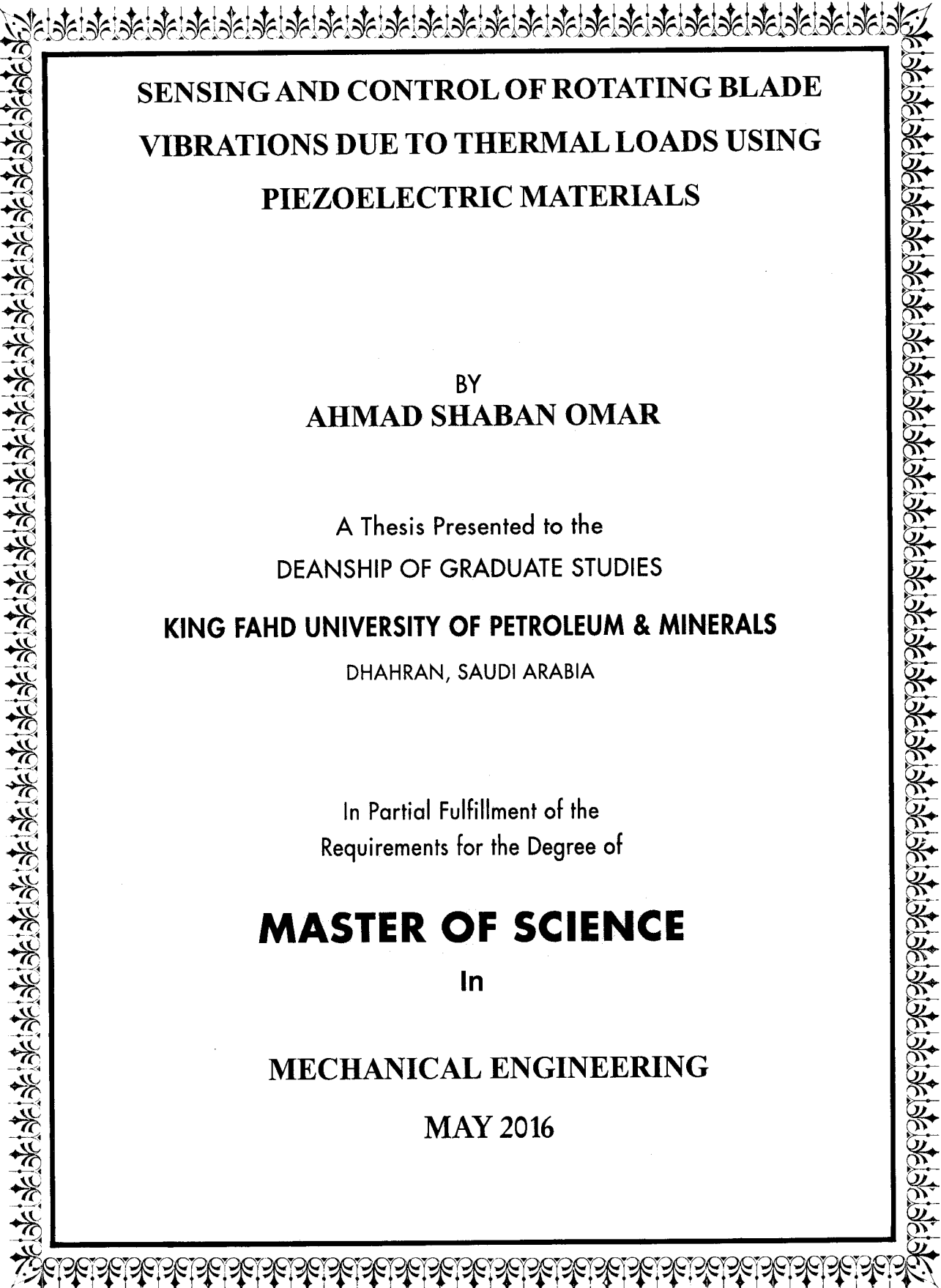
Sensing and Control of Rotating Blade Vibrations Due to Thermal Loads Using Piezoelectric Materials	العنوان:
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**SENSING AND CONTROL OF ROTATING BLADE  
VIBRATIONS DUE TO THERMAL LOADS USING  
PIEZOELECTRIC MATERIALS**

BY  
**AHMAD SHABAN OMAR**

A Thesis Presented to the  
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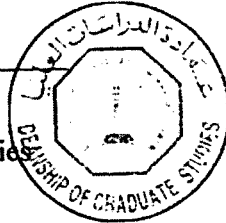
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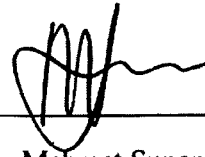
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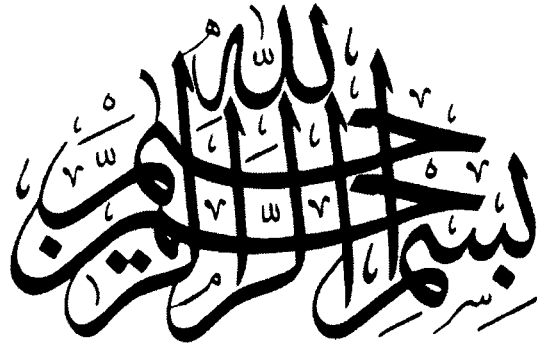
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*To my Beloved*

*Parents, Brother, and Sisters*





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*"In the name of Allah, the Most Beneficent and Most Merciful"*

First of all, I would like to thank almighty Allah for this achievement. He provided me with his blessings throughout all my life and gave me the health, patience and strength to reach this level of knowledge and accomplish this work. I pray to almighty Allah to make this thesis beneficial knowledge "on the day when neither wealth nor sons shall benefit anyone, except for him who comes before Allah with a pure heart".

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# CHAPTER 1

## INTRODUCTION

---

Thermal analysis for temperatures, stresses, and/or displacements has always been a focus of interest in scientific communities due to involvement of thermal field in vastly different areas [1]. In particular to thermally-induced vibrations, some examples include applications of high temperatures and heat in turbo machinery to produce power.

Beams are basic, but important structural elements, because many systems can be simply modeled as such. Hence, they have been the subject of many investigations in various mechanical and structural studies; a robot arm, a turbine blade, and an airplane wing can be modeled as a beam, to name a few [2].

Thermopiezoelectric or piezothermoelastic materials can be used in vibration sensing and control because of the thermopiezoelectric phenomenon, which couples thermal, mechanical, and electrical fields together. Hence, these materials have found many applications in robotics, aerospace, turbomachinery, microelectromechanical systems (MEMS), etc. [3].

The finite element method (FEM) is a robust numerical approach used in the analysis of many modern systems. It divides a system into small parts, called finite elements, and then assembles them to solve for temperatures, displacements. etc., on the



global system. The solution is then post-processed to obtain dependent variables such as strains and stresses.

## 1.1 LITERATURE REVIEW

In 1880, two French physicist brothers, Jacques and Pierre Curie were the first who discovered the piezoelectric effect [4], [5]. Piezoelectricity basically defines a relationship between electric and mechanical stress/strain fields in certain materials [6]. Specially shaped crystals of natural minerals, quartz in particular, were originally used. Nowadays, there are several piezoelectric materials used in various applications including: Lead Zirconate Titanate (PZT), Aluminum Nitride (AlN), Zinc Oxide (ZnO), Quartz (SiO<sub>2</sub>), Poly-Vinylidene Fluoride (PVDF) and others [7]. The manufactured ceramics are the most commonly used piezoelectric materials such as Ceramic B, PZT-2, PZT-4, PZT-5A, PZT-5H, etc., due to their special characteristics, and the piezoelectric materials. The piezoceramics are used as either sensors or actuators [8].

Adaptive structures using piezoelectric materials are usually called smart structures. The piezoelectric materials have been utilized extensively in precision control of dynamical systems. Using piezoelectric materials in active control applications has proved that they are very good in sensing and controlling to reduce vibration and counteract any undesired disturbance.

Thermally induced vibrations of internally heated beams were analyzed by Murozono [9]. Theoretical and experimental results were obtained and compared. Composite beams with interlayer slip operating under thermal loads were examined by Adam et al [10]. The resulting boundary value problem was solved using a mixed closed

form and truncated modal series procedure for the quasi-static and complementary dynamic portions. Structural vibrations due to thermal loads were controlled via piezoelectric pulses by Tauchert and Ashida [11]. Al-Bedoor et al [12] developed a technique for blade vibration measurements in turbo-machinery and jet engines.

Active vibration control of a cantilever beam with a pair of piezoceramic patches was considered by Vasques and Rodrigues [13] where they analyzed velocity response of the beam to a harmonic excitation and an initial displacement field. Sunar and Al-Bedoor [2] investigated the usability of a piezoceramic (PZT) sensor placed in the root of a stationary cantilever beam for measuring structural vibrations during both the transient and steady-state phases.

In another study, free and forced vibration of a laminated variable thickness FGM Timoshenko beam under heat effects was investigated by Xiang and Yang [14]. The beam was imposed upon one dimensional steady heat in its thickness direction and Lagrange interpolation polynomials were utilized as a numerical tool to solve the resulting thermo elastic and dynamic equations. Dynamical response of a cylindrical rod subjected to a non-uniform heat was analyzed by Bertarelli et al [15] through analytical and experimental means. The applied heat causes fast temperature increase in the rod producing longitudinal and flexural vibrations in return. A lightweight piezocomposite actuator was used by Suhariyono et al [16] to attenuate vibrations of a cantilever aluminum beam. A digital PID control process was implemented in the active vibration suppression system.

The study of a cantilever beam with a piezoceramic actuator and sensor through a pole placement control of multimodal vibration suppression was presented by Sethi and Song [17]. The spectral element model for the axially moving plates subjected to thermal loadings was developed by Lee and Kwon [18]. Numerical results revealed that dynamic response of the spectral element model for an axially moving uniform plate neglecting its length and considering the plate as one finite element component was very accurate. Vibration and stability studies of a composite beam under thermal and magnetic fields were undertaken by Wu [19] where various results including those displaying phenomena of beats and resonance were shown.

Formulation for analyzing piezothermoelastic fibers compound shell design with attested actuators and piezoelectric sensors was developed by Roy et al developed [20]. A simply-supported beam subjected to step heat input, sudden heating and dynamic stresses was studied by Marakala et al [21]. They used the finite element method for the solution to thermally induced vibrations. They found out that the displacement increases while the frequency of oscillation decreases. Displacement control of a compound cylinder comprising of a thermoelastic inner layer in touch with the outer layer was considered by Tauchert and Howard [22]. Voltage was applied to the outer surface of the cylinder in order to repress the thermally-induced radial displacement.

An Euler-Bernoulli beam of large rotation was analyzed by Shen et al [23] for the coupled effects of thermal and structural loads. Thermally induced vibration of a cantilever beam was studied by Kidawa-Kukla [24] with an assumption of the surface of the beam acting as a periodically time-varying heat source. Thermal stresses produced by the changing of the beam temperature led to displacements in the beam.

Thermal impact on dynamics of a damped cantilever Timoshenko beam of temperature-dependent elastic modulus was analyzed theoretically by Gu et al [25]. Temperature gradient induced thermal force that helped to determine the thermal effect.

Malgaca et al studied the control behaviors of rotating blades vibrations through piezoelectric materials at the root [26]. Smart materials were used by Kerboua et al [27] to reduce and control the vibration of cantilever beams. Optimal design and location of piezoelectric transducers were studied by applying the finite element method. Romaszko et al [28] analyzed forced vibrations of a homogeneous cantilever beam subjected to kinematic excitation with a moving holder via the application of a vision method.

## **1.2 OBJECTIVES**

The main objective of this thesis is the sensing and control of thermally generated responses of stationary and rotating beams by the use of piezoelectric materials. This study is important for mechanical/structural elements subjected to heat loads that cannot be ignored. These heat inputs alone may result in thermally induced vibrations or may add up to already existing mechanical vibrations from other sources. Monitoring and controlling vibrations due to thermal and other effects are very important for the safe functioning of industrial systems operating under various disturbances.

### **1.3 LAYOUT OF THESIS**

In the first part of this thesis in chapter 2, simply supported and cantilever stationary blades (beams/plates) are subjected to the step load of uniform heat, resulting in time varying temperature and displacement fields in the blade. Temperature and lateral displacement fields are computed analytically then by finite element methods. Analytical and finite element results are analyzed and compared with each other for verification. Lateral displacements of the blade are then sensed and controlled by piezoelectric materials bonded to the beam through a feedback control scheme.

In the second part of the thesis in chapters 3 and chapter 4, the cantilever blade mounted with piezoelectric materials is given rotation and then the rotating system is considered for thermally induced vibration sensing and control. The feedback controller designed for the stationary blade is used for the rotating blade control with some feedback gain modifications.

## CHAPTER 2

# THERMALLY-INDUCED VIBRATION ANALYSIS FOR STATIONARY BLADES

---

In this chapter, analytical solution of Fourier heat conduction equation given below is presented and solved for temperature distribution in a beam (blade) due to a thermal load. The resulting temperature distribution is used to obtain lateral displacements of the beam by employing the beam deflection equation provided in the following section. The same beam is also modeled by ANSYS finite element program via transient thermal analysis and solved for temperature, which is utilized through transient structural analysis to obtain lateral displacements of the beam. The analytical and finite element results are then assessed and compared with each other.

## 2.1 Analytical Method

### 2.1.1 Temperature Distribution

The heat conduction is one of three modes by which the heat transfer occurs. For an isotropic solid, the heat conduction or diffusion equation is given by [29]:

$$k(\nabla^2 T) + Q = (\rho c) \frac{\partial T}{\partial t} + (\rho c) \frac{\partial(uT)}{\partial x} \quad (2.1)$$

where:

$k$ : thermal conductivity (W/mK),

$\nabla^2$ : The gradient vector,

$T$ : Solid temperature (K),

$Q$ : Heat input rate per unit volume ( $W/m^3$ ),

$\rho$ : Material density ( $kg/m^3$ ),

$c$ : Specific heat ( $J/kg.K$ ), and

$t$ : time (sec).

Equation 2.1 is solved to determine the beam temperature  $T$  for various initial and boundary conditions. The initial conditions specify the initial temperature distribution of the beam, which is taken to be constant for most problems at time zero, i.e.  $T(x,0) = T_\infty$ , where,  $T_\infty$  is the free air stream temperature caused by the beam. As for the boundary conditions, there are several cases, one of which is the surface insulation. This case requires that there is no heat flux, i.e.  $q_h=0$ , across the insulated surface along a direction of  $x$ . This is mathematically expressed as  $\frac{\partial T}{\partial x} = 0$ . While in other cases, there is a presence of convective heat transfer, i.e.  $q_h = h_c(T - T_\infty)$ , where  $h_c$  is the convective heat transfer coefficient. This is shown through substituting in the heat conduction law:

$$q_h = -k \frac{\partial T}{\partial x} \quad (2.2)$$

By using linear Lagrange interpolation function, the finite element equation for temperature distribution upon the thickness of the plate when the beam is under effect of sudden heating on one side and insulated on other surface is as follows [30]:

$$\frac{kA}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} + \frac{\rho c A l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \end{Bmatrix} = \frac{\dot{q} A l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \quad (2.3)$$

However, for the plate affected by sudden heating on one surface and naturally uniform convection on the other sides of the beam is given by

$$\begin{aligned} & \left\{ \frac{kA}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & h_c A \end{bmatrix} \right\} \begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix} + \frac{\rho c A l}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} \dot{T}_1 \\ \dot{T}_2 \end{Bmatrix} \\ & = \frac{\dot{q} A l}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} + h_c A T_\infty \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \end{aligned} \quad (2.4)$$

where  $T_1$  and  $T_2$  are the nodal temperatures and  $T_\infty$  is the ambient temperature. The 2<sup>nd</sup> vector on the right hand side and 2<sup>nd</sup> matrix on the left hand side are contributed by convection and will be taken into consideration for the last element only.

The global finite element equation for time dependent temperature distribution has the following form:

$$K_{\text{comb}}^e T + K_{\text{cap}}^e \dot{T} = \bar{F}_Q^e \quad (2.5)$$

where  $K_{\text{comb}}^e$  is the convection matrix,  $K_{\text{cap}}^e$  is the element capacitance matrix, and  $\bar{F}_Q^e$  is the force vector. Equation 2.5 must be solved for the variation of temperature in time domains and space to obtain the temperature distribution which can be calculated and solved using the method of finite central difference [31].



The closed form solution for the temperature distribution of the beam is given by [32]

$$T(y, \tau) = \frac{hQ}{k} \left\{ \tau + \frac{1}{2} \left( \frac{y}{h} + \frac{1}{2} \right)^2 - \frac{1}{6} - \frac{2}{\pi^2} \sum_{j=1}^{\infty} \frac{(-1)^j e^{-j^2 \pi^2 \tau}}{j^2} \cos j\pi \left( \frac{y}{h} + \frac{1}{2} \right) \right\} \quad (2.6)$$

where  $y$  is the vertical distance from the neutral axis of the beam,  $h$  is thickness of the beam, and  $\tau$  is a non-dimensional time parameter defined by:

$$\tau = \frac{\kappa t}{h^2} \quad (2.7)$$

The thermal diffusivity  $\kappa$  is given by

$$\kappa = \frac{k}{\rho c} \quad (2.8)$$

## 2.1.2 Thermally-Induced Vibrations of Beams Subjected to External Heat

### Source

Consider a differential element  $dx$ , of a thin beam under the action of external forces and thermal loads.  $T_t$  and  $T_b$  correspond to the temperature on the top and bottom surfaces respectively, while  $F$  is the shear force and  $p$  is the load intensity. The stationary beam is assumed to be an isotropic solid with a uniform cross-section lying in the  $x$  direction. According to [33] the curvature of a uniform beam under the simultaneous actions of applied loads and Euler–Bernoulli’s heat input can be expressed as:

$$M + M_T = EI \left( \frac{\partial^2 v}{\partial x^2} \right) \quad (2.9)$$

where  $M$  is the internal bending moment,  $M_T$  is the thermal moment which is due to temperature gradient across the thickness of the beam,  $I$  is the moment of inertia of the beam cross section, and  $v$  is the lateral displacement in the  $x$  direction.

The thermal moment acts as a forcing function for the beam exposed to heat flux on one surface and insulated on the other sides is given as

$$M_T = \int_A E \cdot \alpha \cdot \Delta T \cdot z \cdot dA \quad (2.10)$$

where  $\Delta T$  is the change in the temperature.

The total thermal moment across the section is found by calculating and summing up the thermal moment at uniform intervals from the top to bottom surfaces of the beam. There is no temperature variation along the length of the beam, which means that the temperature is independent of  $x$ , hence,  $M_T = M_T(t)$ .

Differentiating of equation 2.9 twice with respect to  $x$ , the resulting expression is

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M_T}{\partial x^2} = \frac{\partial^2}{\partial x^2} \left( EI \frac{\partial^2 v}{\partial x^2} \right) \quad (2.11)$$

According to D'Alembert's principle the applied loads will be equated to the negative of the inertia force as follows:

$$\frac{\partial^2 M}{\partial x^2} = -\rho A \left( \frac{\partial^2 v}{\partial t^2} \right) \quad (2.12)$$

where  $\rho$  is the density of the beam, and  $A$  is the cross sectional area.

Using equation 2.12 in equation 2.11 will yield in the governing equation for lateral displacement (deflection) of a beam in the transverse direction in the presence of thermal moment as

$$\frac{\partial^2 M_T}{\partial x^2} = EI \frac{\partial^4 v}{\partial x^4} + \rho A \frac{\partial^2 v}{\partial t^2} \quad (2.13)$$

For vibrations due to the above thermal load, the stationary beam is assumed to be an isotropic solid with a uniform cross-section lying in the  $x$  direction. Hence, the governing equation for the lateral beam displacement  $v$  is expressed as [34]:

$$(EI) \frac{\partial^4 v}{\partial x^4} + (\rho A) \frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 M}{\partial x^2} \quad (2.14)$$

where  $E$  is the beam's modulus of elasticity,  $I$  is second area-moment,  $A$  is cross-sectional area,  $\rho$  is mass density, and  $M$  is moment caused by thermal effects generated within the beam.

The solution of this equation again requires initial and boundary conditions for the displacement field. The beam is considered to be initially at rest with simply supported boundary condition for the previous analytical study. The non-dimensional thermal moment is given by

$$m(\xi, \tau) = \frac{\pi^4 k M}{192 Q E I \alpha} \quad (2.15)$$

where  $\alpha$  is thermal expansion coefficient, and the non-dimensional parameter  $\xi$  is defined as the ratio of variable  $x$  to length of the beam  $L$

$$\xi = \frac{x}{L} \quad (2.16)$$

The non-dimensional lateral deflection  $V$  is defined in terms of physical lateral deflection  $v$  by

$$V(\xi, \tau) = \frac{\pi^4 kv}{192Q\alpha L^2} \quad (2.17)$$

Another non-dimensional parameter  $B$  is introduced as

$$B = \frac{h}{L\sqrt{\kappa}} \left( \frac{EI}{\rho A} \right)^{1/4} \quad (2.18)$$

where  $h$  is the height or thickness of the rectangular beam.

The non-dimensional lateral deflection is then found as

$$V = V_{st} - \sum_{n=1,3,5}^{\infty} \frac{\sin n\pi\xi}{n^3\pi^3} \left[ \frac{\pi^2}{8B^2n^2} \sin n^2\pi^2 B^2\tau - \sum_{j=1,3,5}^{\infty} \frac{e^{-j^2\pi^2\tau} + (j/nB)^2 \sin n^2\pi^2 B^2\tau - \cos n^2\pi^2 B^2\tau}{j^4 + n^4 B^4} \right] \quad (2.19)$$

where  $V_{st}$  is the static non-dimensional deflection solution given by

$$V_{st}(\xi, \tau) = -\frac{m_T}{2} (\xi^2 - \xi) \quad (2.20)$$

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